Digital Finite Time Control of Robotic Arm by Implicit Discretization

Track 18: ASME International Undergraduate Research and Design Expo

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Project Objectives and Goals

The objectives and goals of this project:

- Explore the application of implicit discretization to terminal sliding mode control (TSMC)
- Simulate a robotic arm in MATLAB simulation environment to demonstrate the effectiveness of the proposed method

Background

- Finite time control (FTC)
  - Achieve a desired state within a finite time window with minimal error
- Types of FTC:
  - Sliding mode control (terminal, integral, adaptive, etc.)
  - Fuzzy/neural network technique
- Finite time stability
  - The origin is Lyapunov stable
  - The trajectory converges within finite time
- Digital FTC
  - Discrete time FTC should be implemented for digital control
- Technical gap
  - Transitioning form continuous to discrete FTC is not straightforward
    - Chattering effect
    - Instability
- Chattering
  - A system oscillates back and forth with high frequency near the desired state

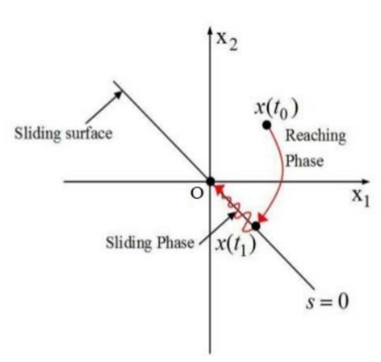

Figure 1. The Chattering Effect [2]

  - Chattering is an even bigger issue for digital FTC due to lack of smoothness
  - Chattering can be caused by unmodeled dynamics or discrete time implementation
- Chattering suppression methods in continuous time systems [3]:
  - Observer-based chattering suppression
  - State-dependent gain method
  - Equivalent-control dependent gain method
- Applications
  - High precision control
    - Robotics (manufacturing)
      - Aerospace (satellite altitude control, missile guidance)
  - FTC for multiple-agent system
    - Mission with multiple subobjectives

Methods & Procedures

- Sliding mode control (SMC) is a type of FTC where a system slides onto a desired surface [4]
  - Control law and switching function
- General equations for SMC that represent the single dimension motion of a unit of mass [5]:

$$\dot{x}_1 = x_2 \qquad x_1(0) = x_{10}$$
$$\dot{x}_2 = u + f(x_1, x_2, t) \qquad x_2(0) = x_{20}$$

- u is the control force
- $f(x_1, x_2, t)$ is the disturbance

SMC Example

- Digital Implementation is a crucial aspect of being able to represent, display and utilize SMC.
- To digitally implement the SMC methods a scaled pendulum will be discussed
- The scaled pendulum from [4] will be used where:

$$\ddot{y} = -sin(y) + u$$
$$s = \dot{y} + y$$
$$u = -\dot{y} - 2sgn(s)$$

- y is the angular position
- u is the control
- s is the switching function
- Figure 2 represents the continuous and explicit discretization graph of angular position (x_1) with continuous time, a step size of 0.05 and a step size of 0.1.

- With a step size of 0.05 the system begins to chatter
- With a step size of 0.1 the system does not converge upon the desired state
- Figure 3 shows the angular position and velocity vs. time when using the implicit Runge-Kutta method with a step size of 0.05.
- Unlike with the explicit method, the implicit method begins to converge, demonstrating how the implicit method is more effective in eliminating chattering.

TSMC Robotic Arm

- TSMC is a type of SMC that guarantee's a system's convergence within a finite time
- Based off the paper from [8] three methods of SMC of a two-joint robotic arm will be simulated and compared:
  - Nonsingular fixed-time TSMC (NFTSMC)
  - Robust fixed-time SMC (RFMC)
  - Fast TSMC (FTSMC)
- For these examples ode45 from MATLAB will be used with a step size of 0.1.
- Figure 4 and 5 respectively show joint 1 and joint 2 vs. time.
- Figure 6 and 7 respectively show the join 1 error and the joint 2 error vs. time.
- NFTSMC experiences the smallest integrated absolute error (IAE) with a value of 0.216.

Data and Results

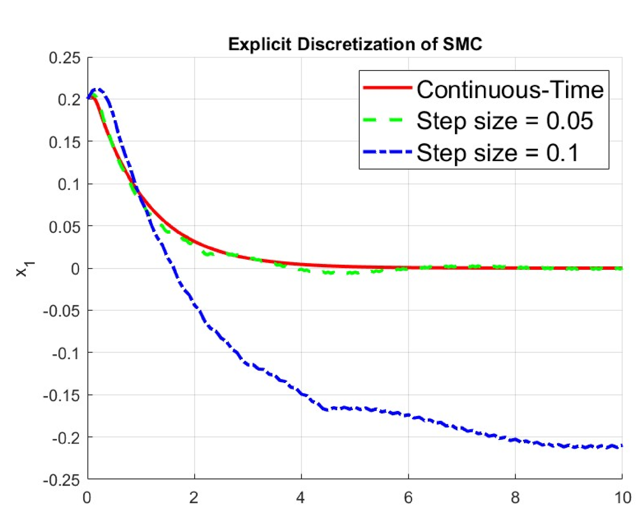

Figure 2. Explicit discretization of SMC of x_1 vs. time

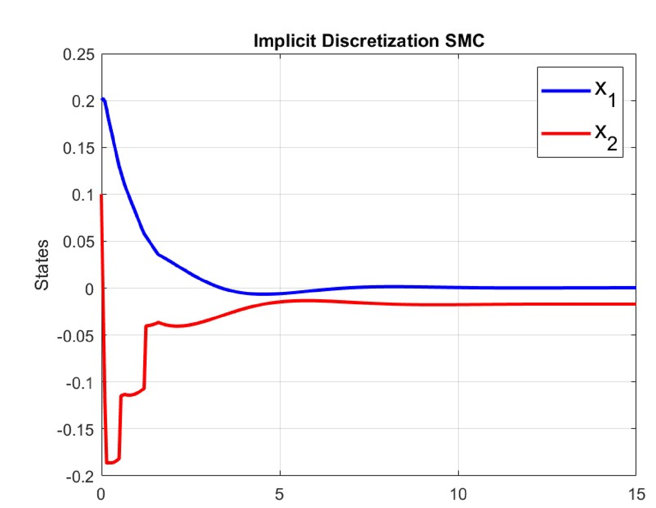

Figure 3. Implicit discretization of SMC of x_1 and x_2 vs. time

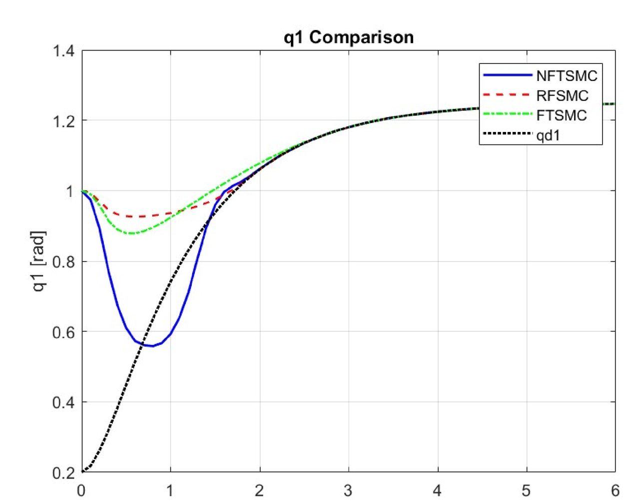

Figure 4. Joint 1 vs. time

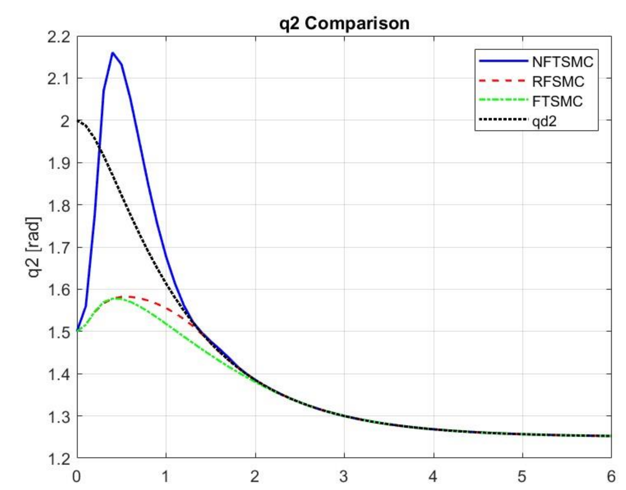

Figure 5. Joint 2 vs. time

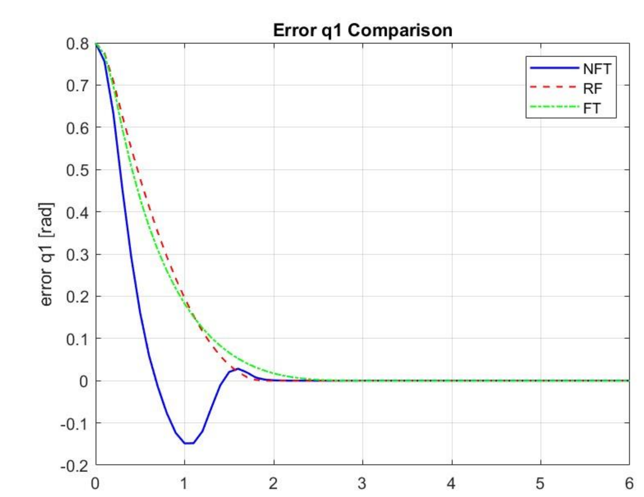

Figure 6. Joint 1 error vs. time

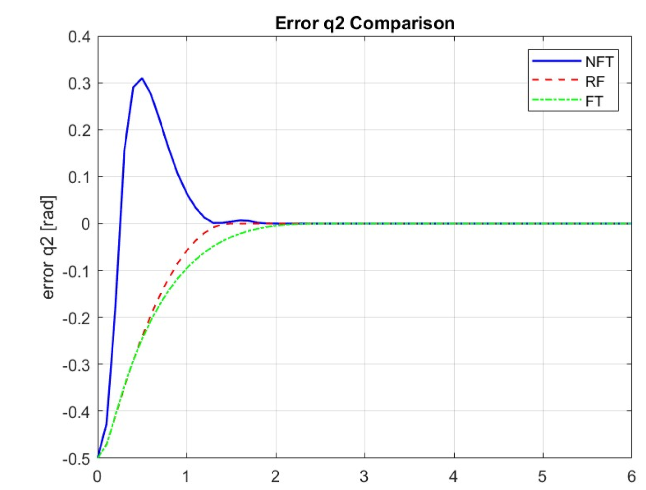

Figure 7. Joint 2 error vs. time

Conclusions

- Preliminary simulations agree with the fact that implicit discretization allows for less chattering than explicit discretization as can be seen in Figure 2 and Figure 3
- A robotic arm example has been successfully simulated and demonstrates that the NFTSMC is a viable option to use for further testing and simulations related to implicit discretization and robotic arms

Future Studies / Recommendations

These are preliminary results related to the robotic arm simulation. For future studies:

- Specific implicit methods need to be explored and compared to explicit methods
- Different initial conditions should be explored
- This is a two-joint robotic arm, consider exploring a three-joint robotic arm example
- Test the simulation on a robotic arm testbed to confirm the results of the simulations

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