Realizing Digital Finite Time Control by Implicit Discretization

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Finite Time Control

- Finite time control (FTC)
  - Achieve a desired state within a **finite time window**
- Types of FT control
  - Sliding mode control (terminal, integral, adaptive, etc.)
  - Fuzzy/neural network technique
- Finite time stability
  - The origin is Lyapunov stable
  - The trajectory converges within finite time, which is characterized by settling time function $T(x_0)$
    - $T(x_0) = \min\{t \in \mathbb{R}_+ : x(t) = 0\}$

Digital Finite Time Control

- Digital FTC
  - FTC technique should be digitally implemented
- Technical Gap
  - Transitioning from continuous to discrete FTC is not straightforward
    - Chattering effect
    - Instability
- **Chattering**
  - A system oscillates back and forth with high frequency near the desired state
  - Chattering is even bigger issue for digital FTC due to the lack of smoothness
  - Chattering can be caused by unmodeled dynamics or discrete time implementation
- Chattering suppression methods in CT [3]
  - Observer-based chattering suppression
  - State-depending gain method
  - Equivalent-control dependent gain method

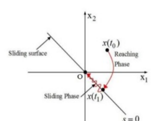

Figure 1. The chattering effect [2]

Applications

- High precision control
  - Robotics (manufacturing)
  - Aerospace (satellite attitude control, missile guidance)
- FTC for multi-agent system
  - Mission with multiple subobjectives
- Digital implementation of FTC poses a big challenge

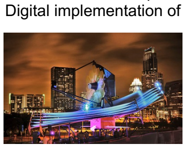
Figure 2. James Webb Space Telescope (credit: NASA JWST)


Figure 3. Manufacturing robotic arm for semiconductor

Sliding Mode Control

- Sliding mode control (SMC) is a type of FT control where a system slides onto a desired surface [4]
  - Control law and the switching function
- General equations for SMC that represent the single dimensional motion of a unit mass [5]

$$\dot{x}_1 = x_2 \qquad x_1(0) = x_{10}$$
$$\dot{x}_2 = u + f(x_1, x_2, t) \qquad x_2(0) = x_{20}$$

  - u is the control force
  - $f(x_1, x_2, t)$ is the disturbance

SMC Example

- Digital implementation is a crucial aspect of being able to represent, display, and utilize SMC
- To digitally implement the SMC methods a scaled pendulum will be discussed as can be seen in figure 4.
- The scaled pendulum from [2] will be used where:

$$\ddot{y} = -\sin(y) + u$$
$$s = \dot{y} + y$$
$$u = -\dot{y} - 2\,\mathrm{sgn}(s)$$

- y is the angular position
- u is the control
- s is the switching function

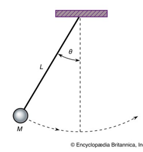

Figure 4. Scaled Pendulum [6]

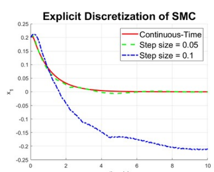

Figure 5. Continuous and Explicit Discretization graph of x_1

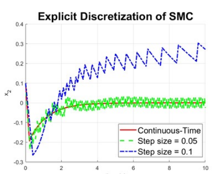

Figure 6. Continuous and Explicit Discretization graph of x_2

- The continuous SMC, explicit method with a step size of 0.05 and with a step size of 0.1 are shown in figures 5 and 6
  - Figure 5 graphs x_1, which represents the angular position, as a function of time
  - Figure 6 graphs x_2, which represents the angular velocity, as a function of time
- Figure 5 and 6 demonstrate the technical gap that occurs when transitioning from continuous to discrete time and the significance of the step size selected
  - With a step size of 0.05 the system begins to chatter
  - With a step size of 0.1 the system does not converge upon the desired state in Figure 5 and in Figure 6 the system does not converge and is chattering

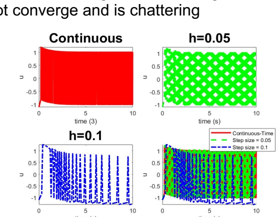

Figure 7. Continuous and Explicit Discretization graph of u

- Figure 7 graphs the control, which changes based off the switching function, of the continuous and explicit discretization methods as a function of time
  - In the continuous method the switching function experiences a high frequency input switching, chattering, to achieve stability
  - With a step size of 0.05, the explicit method still experiences high frequency input switching, but less than that of the continuous method
  - With a step size of 0.1 the system experiences a lower frequency input switching and as a result is unstable

Ongoing Study

- The ongoing study is determining ways to implement implicit discretization methods to decrease the chattering in discrete time
- The implicit method using the example from [2] can be seen in figure 8
- Unlike the explicit method, the implicit method performs a correction after the discretization
- The explicit method may become unstable and is more likely not to converge
- The implicit method requires more computational power than the explicit method

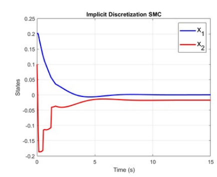

Figure 8. Implicit Discretization of SMC

- In figure 8:
  - The Runge-Kutta method is implemented, which is an implicit method
  - Both x_1 and x_2 are shown in the figure 8, which are the states of the SMC method
  - Once the system converges it experiences far less chattering than the explicit methods from figures 5, 6, and 7
  - Confirms that the implicit method results in less chattering than the explicit method
- The explicit method :

$$x_{k+1} = x_k + hu_k + hd_k$$
$$u_k = -\mathrm{sgn}(x_k)$$

- The implicit method [7]:

$$x_{k+1} = x_k + hu_k + hd_k$$
$$\tilde{x}_{k+1} = x_k + hu_k$$
$$u_k = -\lambda_{k+1}$$
$$\lambda_{k+1} \in \mathrm{sgn}(\tilde{x}_{k+1})$$

Future Study

- Develop and compare digital implementation methods for different implicit discretization methods.
- Utilize multiple examples to demonstrate the digital implementation methods.
- Apply the digital implementation methods to a 3-axis robotic arm application.

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